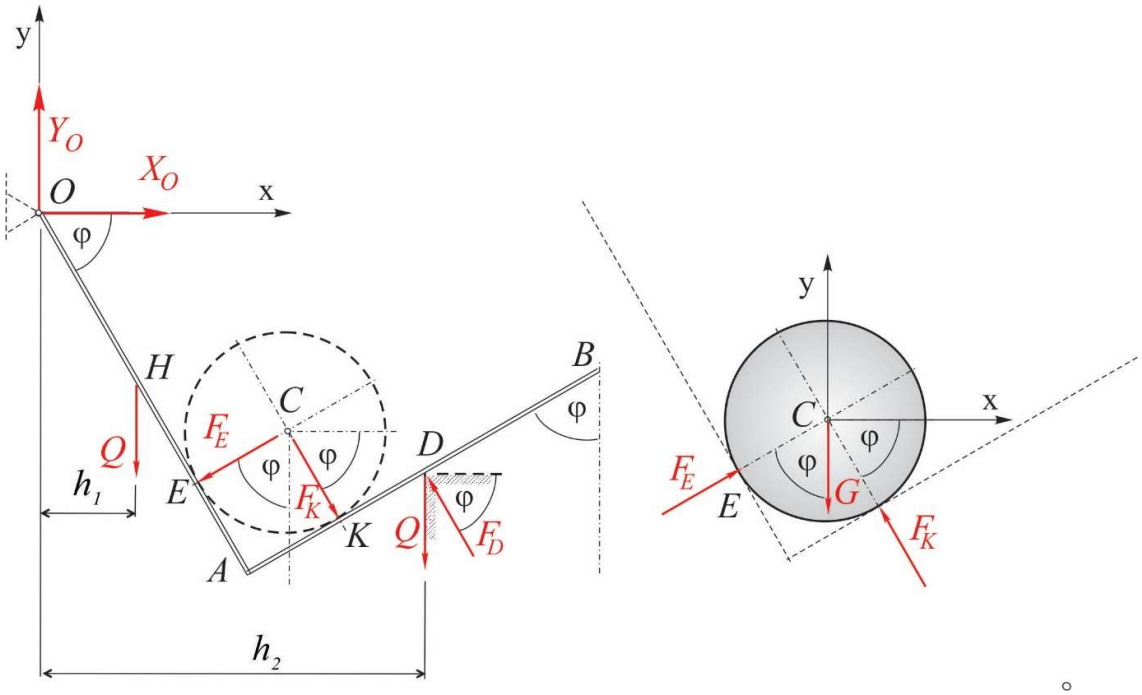
# 1. Задатак

**1.1** Означавање свих сила тежине и свих реакција везе **(9x1=9)**




**1.2** Услови равнотеже за диск **(2x3=6)**

$$\sum X_i = 0; \quad F_E \sin\varphi - F_K \cos\varphi = 0; \qquad \textbf{(3)}$$

$$\sum Y_i = 0; \quad F_E \cos\varphi + F_K \sin\varphi - G = 0; \qquad \textbf{(3)}$$

**1.3** Услови равнотеже за штап **(2x5+7=17)**

$$\sum X_i = 0; \quad X_O - F_E \sin\varphi + F_K \cos\varphi - F_D \cos\varphi = 0; \qquad \textbf{(5)}$$

$$\sum Y_i = 0; \quad Y_O - Q - F_E \cos\varphi - F_K \sin\varphi - Q + F_D \sin\varphi = 0; \qquad \textbf{(5)}$$

$$\sum M_O = 0; \quad -Q \cdot h_1 - F_E \cdot 3R - F_K \cdot R - Q \cdot h_2 + F_D \cdot 2R = 0; \qquad \textbf{(7)}$$

**1.4** Одређивање кракова сила **(2x2=4)**

$$h_1 = \overline{OH}\cos\varphi = 2R\frac{1}{2} = R \qquad \textbf{(2)}$$

$$h_2 = \overline{OA}\cos\varphi + \overline{AD}\sin\varphi = 4R\frac{1}{2} + 2R\frac{\sqrt{3}}{2} = R\left(2+\sqrt{3}\right) \qquad \textbf{(2)}$$

**1.5** Одређивање реакција веза **(2x1+3+2x2=9)**

$$F_K = F_E tg\varphi = \sqrt{3}F_E$$

$$\underline{F_K = \frac{\sqrt{3}}{2}G = 4\sqrt{3}\ \left[kN\right]} \qquad \textbf{(1)}$$

$$F_E\cos\varphi + \sqrt{3}F_E\sin\varphi = G$$

$$\frac{1}{2}F_E + \sqrt{3}F_E\frac{\sqrt{3}}{2} = G$$

$$\underline{F_E = \frac{G}{2} = 4\ \left[kN\right]} \qquad \textbf{(1)}$$

$$F_D = \frac{1}{2R}\left(Q\cdot R + F_E\cdot 3R + F_K\cdot R + Q\cdot R\left(2+\sqrt{3}\right)\right) =$$

$$= \frac{1}{2}\left(G + \frac{G}{2}3 + \frac{\sqrt{3}}{2}G + G\left(2+\sqrt{3}\right)\right)$$

$$\underline{F_D = \frac{9+3\sqrt{3}}{4}G = 28.39\ \left[kN\right]} \qquad \textbf{(3)}$$

$$\mathrm{X}_O = F_E\sin\varphi - F_K\cos\varphi + F_D\cos\varphi = \frac{\sqrt{3}}{2}\frac{1}{2}G - \frac{1}{2}\frac{\sqrt{3}}{2}\mathrm{G} + \frac{1}{2}\frac{9+3\sqrt{3}}{4}G$$

$$\underline{\mathrm{X}_O = \frac{9+3\sqrt{3}}{8}G = 14.20\ \left[kN\right]} \qquad \textbf{(2)}$$

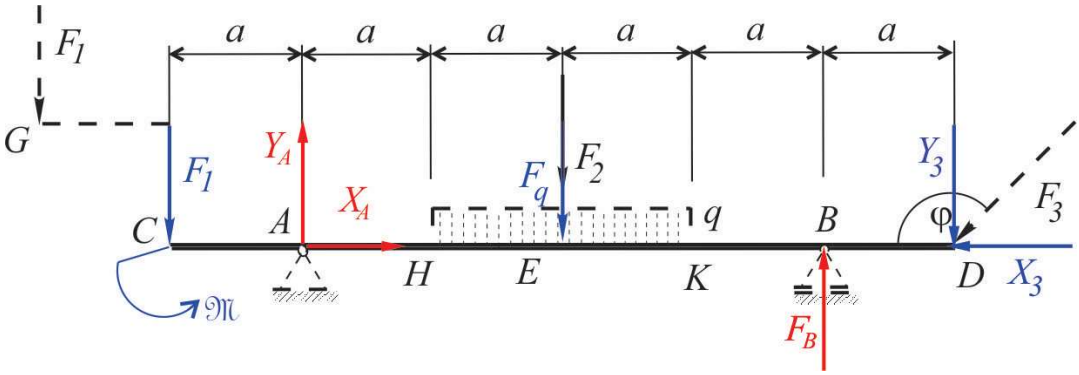
$$\mathrm{Y}_O = Q + F_E\cos\varphi + F_K\sin\varphi + Q - F_D\sin\varphi =$$

$$= G + \frac{1}{2}\frac{1}{2}G + \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}G + G - \frac{\sqrt{3}}{2}\frac{9+3\sqrt{3}}{4}G$$

$$\underline{\mathrm{Y}_O = -\frac{9\sqrt{3}-15}{8}G = -0.59\ \left[kN\right]} \qquad \textbf{(2)}$$

## 2. Задатак

**2.1** Увођење реакција веза **(2)**




**2.2** Увођење евквивалентних дејстава **(4x0.75=3)**

$$\mathcal{M} = F_1 a = 2\ \left[kNm\right], \qquad \textbf{(0.75)}$$

$$F_q = q \cdot 2a = 4\ \left[kN\right], \qquad \textbf{(0.75)}$$

$$X_3 = F_3\, sin\, 45^0 = 2\ \left[kN\right], \qquad \textbf{(0.75)}$$

$$Y_3 = F_3\, cos\, 45^0 = 2\ \left[kN\right]; \qquad \textbf{(0.75)}$$

**2.3** Тачни услови равнотеже **(3+4+4=11)**

$$\sum X_i = 0; \quad \mathrm{X}_A - \mathrm{X}_3 = 0; \qquad \textbf{(3)}$$

$$\sum Y_i = 0; \quad -F_1 + Y_A - F_q - F_2 + F_B - Y_3 = 0; \qquad \textbf{(4)}$$

$$\sum M_A = 0; \quad \mathcal{M} + F_1 \cdot a - F_q \cdot 2a - F_2 \cdot 2a + F_B \cdot 4a - Y_3 \cdot 5a = 0; \qquad \textbf{(4)}$$

**2.4** Тачно израчунате реакције веза **(2+1.5+0.5=4)**

$$F_B = \frac{1}{4a}\left(-\mathscr{M} - F_1 \cdot a + F_q \cdot 2a + F_2 \cdot 2a + Y_3 \cdot 5a\right) = \frac{1}{4}\left(-2-2+8+2+10\right);$$

$$\underline{F_B = 4\ [kN];} \qquad \textbf{(2)}$$

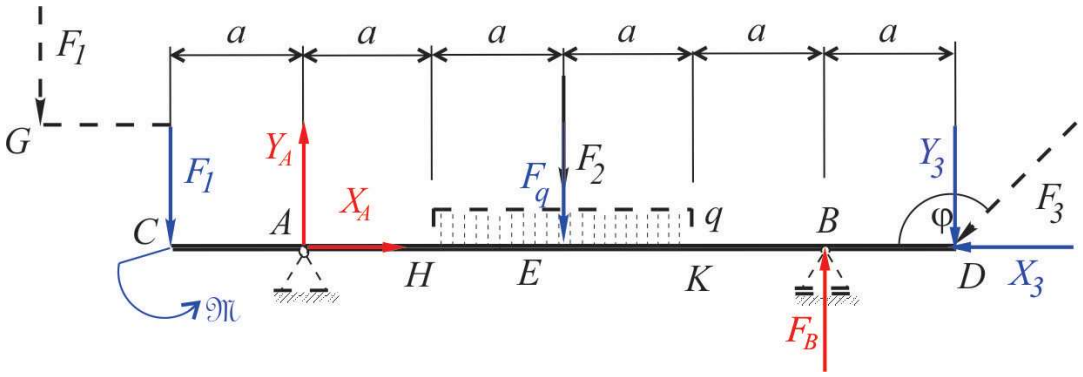
$$Y_A = F_1 + F_q + F_2 - F_B + Y_3 = 2+4+1-4+2;$$

$$\underline{Y_A = 5\ [kN];} \qquad \textbf{(1.5)}$$

$$\underline{\mathrm{X}_A = \mathrm{X}_3 = 2\ [kN];} \qquad \textbf{(0.5)}$$

**2.5** Тачно израчунате вредности трансверзалних сила

**(1+1+0.5+3+3+1+0.5+1+1=12)**




$$F'_{tC} = 0;$$
$$F''_{tC} = -F_1 = -2\ [kN];$$ **(1)**

$$F'_{tA} = -F_1 = -2\ [kN];$$
$$F''_{tA} = -F_1 + Y_A = -2+5 = 3\ [kN];$$ **(1)**

$$F_{tH} = -F_1 + Y_A = -2+5 = 3\ [kN];$$ **(0.5)**

$$F_{tHE}\left(0 \le x \le 1\right) = -F_1 + Y_A - qx = 3 - 2x\ [kN];$$ **(3)**

$$F_{tEK}\left(1 \le x \le 2\right) = -F_1 + Y_{A1} - F_2 - qx = 2 - 2x\ [kN];$$ **(3)**

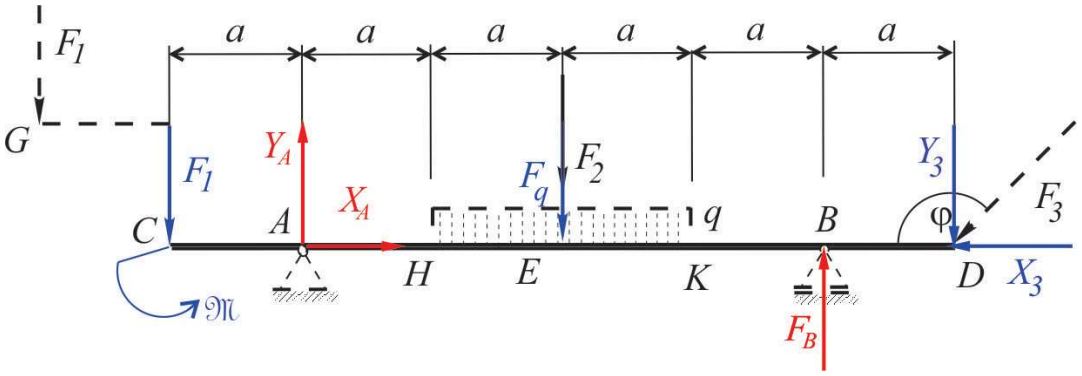
$$F'_{tE} = F_{tHE}\left(x = 1\right) = 1\ [kN];$$
$$F''_{tE} = F_{tEK}\left(x = 1\right) = 2 - 2 = 0;$$ **(1)**

$$F_{tG} = F_{tEK}\left(x=2\right) = 2-4 = -2\ \left[kN\right]; \qquad \textbf{(0.5)}$$

$$F'_{tB} = -F_B + Y_3 = -4+2 = -2\ \left[kN\right];$$
$$F''_{tB} = Y_3 = 2\ \left[kN\right]; \qquad \textbf{(1)}$$

$$F'_{tE} = Y_3 = 2\ \left[kN\right];$$
$$F''_{tE} = 0; \qquad \textbf{(1)}$$

**2.6** Тачно израчунате вредности момента савијања **(6x1+2x3=12)**




$$M'_{fC} = 0\ \left[kNm\right];$$
$$M''_{fC} = -\mathscr{M} = -2\ \left[kNm\right]; \qquad \textbf{(1)}$$

$$M_{fA} = -\mathscr{M} - F_1 a = -2-2 = -4\ \left[kNm\right]; \qquad \textbf{(1)}$$

$$M_{fH} = -\mathscr{M} - F_1 \cdot 2a + Y_A \cdot a = -2-4+5 = -1\ \left[kNm\right]; \qquad \textbf{(1)}$$

$$M_{fHE}\left(0 \le x \le 1\right) = -\mathscr{M} - F_1\left(x+2a\right) + Y_A\left(x+a\right) - q\frac{x^2}{2} = -1+3x-x^2\ \left[kNm\right]; \qquad \textbf{(3)}$$

$$M_{fEG}\left(1 \le x \le 2\right) = -\mathscr{M} - F_1\left(x+2a\right) + Y_A\left(x+a\right) - F_2\left(x-a\right) - q\frac{x^2}{2} = 2x-x^2\ \left[kNm\right]; \qquad \textbf{(3)}$$

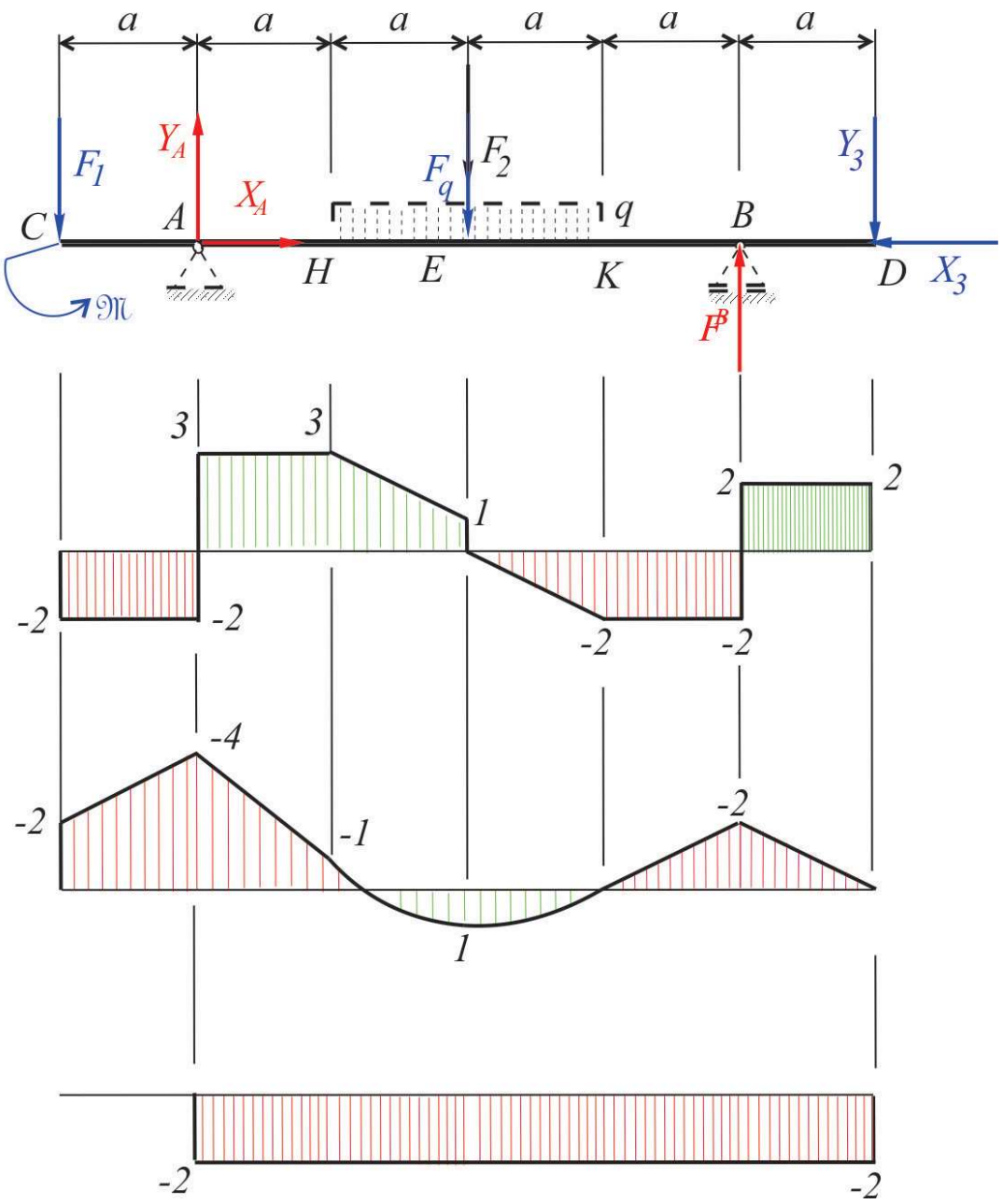
$$M_{fE} = M_{fHE}\left(x=1\right) = M_{fEK}\left(x=1\right) = 1\ \left[kNm\right]; \qquad \textbf{(1)}$$

$$M_{fK} = M_{fEK}\left(x=2\right) = 0\left[kNm\right]; \qquad \textbf{(1)}$$

$$M_{fB} = -Y_3 a = -2\ \left[kNm\right]; \qquad \textbf{(1)}$$

$$M_{fD} = 0;$$

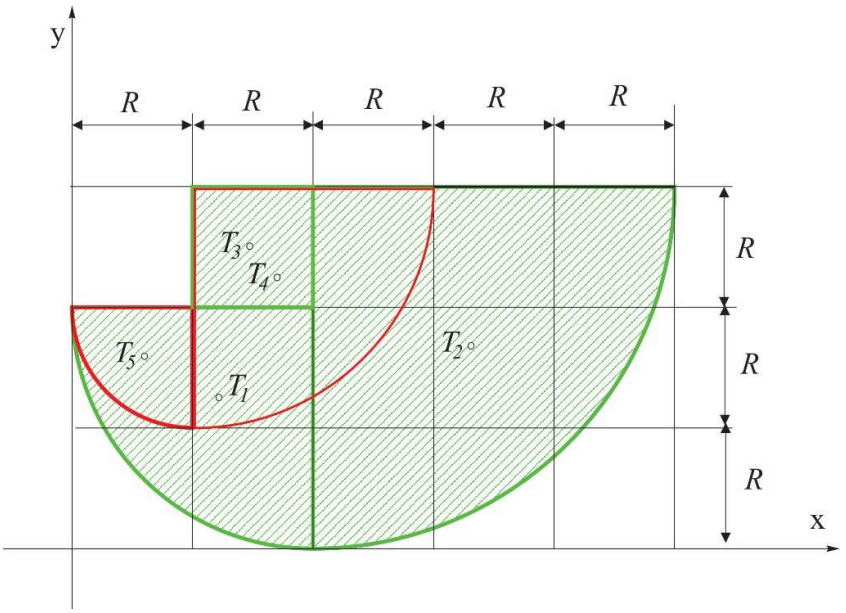
**2.7** Тачно нацртани дијаграми **(6+7+3=16)**



## 3. Задатак

**3.1** Подела на елементарне површине **(5x0.6=3)**

**3.2** Уцртана тажишта елементарних површина **(5x0.6=3)**




**3.3** Тачно израчунате површине **(5x2=10)**

$$A_1 = \frac{(2R)^2\pi}{4} = R^2\pi \approx 3.14 \left[m^2\right]$$

$$A_2 = \frac{(3R)^2\pi}{4} = \frac{9}{4}R^2\pi \approx 7.07 \left[m^2\right]$$

$$A_3 = R \cdot R = R^2 = 1 \left[m^2\right]$$

$$A_4 = \frac{(2R)^2\pi}{4} = R^2\pi \approx 3.14 \left[m^2\right]$$

$$A_5 = \frac{R^2\pi}{4} \approx 0.785 \left[m^2\right]$$

**3.4** Тачно одређене координате површина **(10x2=20)**

$$x_1 = 2R - \frac{4(2R)}{3\pi} = 2R - \frac{8R}{3\pi} \approx 1.15 \left[m\right]$$

$$x_2 = 2R + \frac{4(3R)}{3\pi} = 2R + \frac{4R}{\pi} \approx 3.27 \left[m\right]$$

$$x_3 = 2R + \frac{1}{2}R = \frac{3}{2}R = 1.5 \left[m\right]$$

$$x_4 = R + \frac{4(2R)}{3\pi} = R + \frac{8R}{3\pi} \approx 2.85 \left[m\right]$$

$$x_5 = R - \frac{4R}{3\pi} \approx 0.58 \left[m\right]$$

$$y_1 = 2R - \frac{4(2R)}{3\pi} = 2R - \frac{8R}{3\pi} \approx 1.15 \left[m\right]$$

$$y_2 = 3R - \frac{4(3R)}{3\pi} = 3R - \frac{4R}{\pi} \approx 1.73 \left[m\right]$$

$$y_3 = 2R + \frac{1}{2}R = \frac{5}{2}R = 2.5 \left[m\right]$$

$$y_4 = 3R - \frac{4(2R)}{3\pi} = 3R - \frac{8R}{3\pi} \approx 2.15 \left[m\right]$$

$$y_5 = 2R - \frac{4R}{3\pi} \approx 1.58 \left[m\right]$$

**3.5** Тачно израчунате координате тежишта сложене површине **(2x3=6)**

$$x_T = \frac{x_1A_1 + x_2A_2 + x_3A_3 - x_4A_4 - x_5A_5}{A_1 + A_2 + A_3 - A_4 - A_5} = \frac{\frac{22+21\pi}{4}R^3}{(1+2\pi)R^2}$$

$$\underline{x_T = \frac{1}{4}\cdot\frac{22+21\pi}{1+2\pi}R \approx 3.02 \quad [m]}$$

$$y_T = \frac{y_1A_1 + y_2A_2 + y_3A_3 - y_4A_4 - y_5A_5}{A_1 + A_2 + A_3 - A_4 - A_5} = \frac{\frac{63\pi-74}{12}R^3}{(1+2\pi)R^2}$$

$$\underline{y_T = \frac{1}{12}\cdot\frac{63\pi-74}{1+2\pi}R \approx 1.42 \quad [m]}$$

**3.6** Уцртано тежиште сложене површине **(3)**